

§[1] Complex numbers:

We have already known in the field of real numbers, the equation $x^2 + 1 = 0$ has no solution. In order to permit the solution of this and similar equations, the real number system was extended to the set of complex numbers. Euler (1707-1783) the first mathematician who introduced the symbol i with property that $i^2 = -1$. He also called i as the imaginary unit. A number of the form $a + ib$, where a and b are real numbers, is called complex number.

If we write $z = x + iy$, the z is called a complex variable. x and y are called respectively real and imaginary parts of z . Sometimes we express z as $z = (x, y)$

We also write $R(z) = x, I(z) = y$.

If $x = 0$, i.e. $z = iy$, the z is called pure imaginary number.

The complex conjugate, or briefly conjugate, of $z = x + iy$ is $\bar{z} = x - iy$

for example, conjugate of $-2 - 3i$ is $-2 + 3i$.

It can be easily verified that

$$R(z) = \frac{z + \bar{z}}{2}, \quad I(z) = \frac{z - \bar{z}}{2i}$$

§[2] Equality of complex numbers:

Let us consider two complex numbers:

$$z_1 = x_1 + iy_1, \quad z_2 = x_2 + iy_2$$

Then $z_1 = z_2$ if and only if $x_1 = x_2, y_1 = y_2$.

Note: The phrases "greater than" or "less than" have no meaning in the set of complex numbers.

§[3] Fundamental operations with complex numbers:

(i) Addition: $(a + ib) + (c + id) = (a + c) + i(b + d)$.

(ii) Subtraction: $(a + ib) - (c + id) = (a - c) + i(b - d)$.

(iii) Multiplication: $(a + ib)(c + id) = (ac - bd) + i(bc + ad)$.

(iv) Division: $\frac{a + ib}{c + id} = \frac{(a + ib)(c - id)}{(c + id)(c - id)} = \frac{(ac + bd) + i(bc - ad)}{c^2 + d^2} = \left(\frac{ac + bd}{c^2 + d^2}\right) + i\left(\frac{bc - ad}{c^2 + d^2}\right)$ if $c^2 + d^2 \neq 0$.

§[4] Absolute Value: The absolute value or modulus of a complex number $z = a + ib$

is denoted by $|a + ib|$ and is defined as

$$|a + ib| = \sqrt{a^2 + b^2}$$

Evidently, $|z|^2 = a^2 + b^2 = (a + ib)(a - ib) = z\bar{z}$

$\therefore |z|^2 = z\bar{z}$

Also, $z_1 \cdot z_2 = \bar{z}_1 \cdot \bar{z}_2$ which should be always remembered as a formulae

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